B.Sc Physics (Hons) Part I
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Ques Derive an expression for the gravitational potential at a point (i) outside (ii) on the surface (iii) inside a solid sphere.
(b) Find gravitational field at these points and show it is proportional to the distance from the centre of the sphere for a point inside it.

Ans (a) Potential due to Solid sphere :-

(i) Point outside the sphere :→

Consider a point P outside at a distance r from centre O. Divide the sphere into a large number of thin spherical shells concentric with the sphere of mass m_1, m_2, m_3 ---- etc respectively.

Now potential at a point outside the shell is

$$V = \frac{GM}{r}$$

∴ Potential at P due to all the shell is

$$V = -\left[\frac{Gm_1}{r} + \frac{Gm_2}{r} + \frac{Gm_3}{r} + \cdots\right]$$

$$V = -\frac{G}{r}(m_1 + m_2 + m_3 + \cdots) = -\frac{GM}{r}$$

where M is the mass of Solid sphere. Hence in the case of solid sphere also the whole mass can be supposed to be concentrated at its centre.

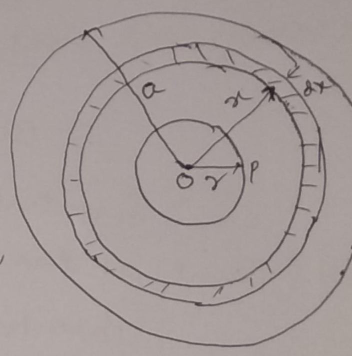
(ii) Point on the surface:

For a point P ~~inside~~ on the surface of the sphere of radius 'a' : $r = a$

$$\therefore V = -\frac{GM}{a}$$

(iii) Point inside the sphere :→

Consider a point P inside the sphere at a distance r from its centre O. Let 'a' & 'ρ' be the radius and density of ~~Earth~~ Sphere. O be the centre and radius OP draw a sphere. The the point P lies on the surface of the Solid sphere of radius 'r' and inside the spherical shell of internal radius 'r' and external radius 'a'.



Volume of the inner Solid sphere $= \frac{4}{3}\pi r^3$

$\therefore$ Mass of the inner Solid sphere $= \frac{4}{3}\pi r^3 \rho$ (2)

Hence potential at P due to inner Solid sphere

$$V_1 = -\frac{\frac{4}{3}\pi r^3 \rho G}{r} = -\frac{4}{3}\pi r^2 \rho G$$

To find the potential due to the outer spherical shell, draw two concentric spheres with radius x and $x+dx$ respectively forming a thin spherical shell of thickness dx.

Now surface area of spherical shell $= 4\pi x^2$

Volume of this shell $= 4\pi x^2 dx$

$\therefore$ Mass of this shell $= 4\pi x^2 \rho dx$

Since the potential at any point within a spherical shell is the same as on the surface.

Potential at P due to shell $= -\frac{4\pi x^2 dx \rho G}{x} = -4\pi x \rho G dx$ ...... (1)

$\therefore$ Potential V_2 at P due to the shell of internal radius r and external radius a is obtained by integrating eqn (1) between the limits $x = r$ and $x = a$

$$\therefore V_2 = \int_r^a -4\pi \rho G x\, dx = -4\pi \rho G \left[\frac{x^2}{2}\right]_r^a$$

$$= -4\pi \rho G \left(\frac{a^2}{2} - \frac{r^2}{2}\right) = -2\pi \rho G (a^2 - r^2)$$

$\therefore$ Total potential at P $= V_1 + V_2$

$$= -\left[\frac{4}{3}\pi r^2 \rho G + 2\pi \rho G (a^2 - r^2)\right]$$

$$= -2\pi \rho G \left(\frac{2}{3} r^2 + a^2 - r^2\right)$$

$$= -\frac{2}{3}\pi \rho G (3a^2 - r^2)$$

$$= -\frac{4}{3}\pi a^3 \rho G \frac{(3a^2 - r^2)}{2a^3}$$

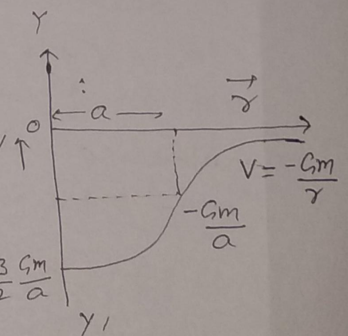
But $\frac{4}{3}\pi a^3 \rho = M$ = Mass of the sphere

$\therefore$ Potential $V = -\frac{GM(3a^2 - r^2)}{2a^3}$

(iv) Point at the centre of sphere. $r = 0$

$\therefore$ Potential $V = -\frac{3}{2}\frac{GM}{a}$

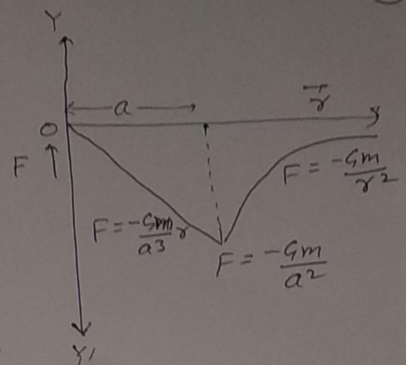
The variation of gravitational potential due to sphere at a point outside it



Ans (b) Gravitational field (attraction)

(i) Point outside the sphere

The gravitational field $F = \frac{-dv}{dx}$



Since the gravitational potential at a point outside the sphere is given by

$$V = -\frac{GM}{r}$$

$$\therefore F = -\frac{dv}{dr} = -\frac{GM}{r^2}$$

(ii) Point on the surface. For a point on the surface of the sphere $r = a$

$$\therefore F = -\frac{GM}{a^2}$$

(iii) Point inside the sphere, ~~For a~~

$$V = -GM\frac{(3a^2 - r^2)}{2a^3}$$

$$\therefore F = -\frac{dv}{dr}$$

$$= -\frac{GM}{\cancel{2}a^3} \times \cancel{2}r$$

$$\therefore F = -\frac{GM}{a^3} r$$

$$F \propto -r$$

Hence the gravitational field at a point inside the solid sphere ~~is~~ is proportional to its distance from the centre.

The variation of gravitational field due to sphere for a point outside the sphere, on the surface and inside it.